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COMPUTER SIMULATION OF EXTRAOCULAR MUSCLE CO-OPERATION: AN EVALUATION

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Abstract—This paper is concerned with specifying how extraocular muscles co-operate in moving the eye. A set of assumptions is described which enable this to be done with enough precision for a computer model of the actions of the extraocular muscles to be set up. The behaviour of the model and its validity are then evaluated.

INTRODUCTION

Krewson (1950) was the first person to produce quantitative estimates of the relationships between rotations of the globe and actions of the muscles. He assumed that they took the mechanical shortest path and calculated their corresponding axes of rotation. To obtain some idea of the mode of action of each of the muscles he considered the projections of the axes of rotation into an eye-centred system of Cartesian axes. As has been conventional since the work of Helmholtz, the system of axes was such that one axis lay along the line of fixation in the primary position and one of the remaining two axes coincided with the line between the centres of rotation of the two eyes. He then considered that the projection onto these axes represented the amount of the forces exerted by each of the muscles that was devoted variously to adduction/abduction, elevation/depression and torsional movements. Because of the number of calculations involved he only considered movements in the horizontal plane. This enabled him to clarify the main actions of the individual muscles. However, his approach, whilst it revealed much about individual muscles, was not so informative about how they co-operate.

Boeder (1961) approached the analysis of extraocular muscle co-operation by calculating the length changes that occur when the muscles follow the shortest path around the globe. He also attempted to provide a more realistic measure of the forces exerted by each of the muscles by multiplying their changes in length by their respective cross-sectional areas. One important conclusion that he formed was that, while the inferior oblique is more contracted than the superior rectus in adduction, it is still the latter which exerts the larger force.

Boeder (1962) went on to consider positions of gaze within a 60 by 60° range. As well as computing the length changes that occur when muscles follow the shortest path over the globe, he also determined the direction in which each muscle would turn the line of fixation in terms of adduction/abduction and elevation/depression rotations. This enabled him to make a number of judicious observations about how the extraocular muscles co-operate. In particular, he considered whether or not it is only the contracting muscles that move the globe. He compared movements from A to B with the return movements from B to A and noticed that the direction of action of the chief shortening muscles was not necessarily the same as that of the chief lengthening muscles, so, if only the contracting muscles moved the eyeball, the movement would be irreversible.

More recently, the purely geometrical calculations of the axes of rotation and changes in lengths of the muscles, have been put into matrix notation by Solomons (1978), who has calculated the adduction/abduction, elevation/depression and torsional action of each of the muscles in primary, secondary and tertiary positions of gaze. The results of these calculations have brought out, *inter alia,* the balanced nature of the torsional effects within pairs of antagonistic muscles. In general, however, whilst this approach has simplified the calculations, it has not by itself revealed anything further about the way in which the muscles co-operate.

This last criticism is especially true if one tries to compute what will happen if some of the muscles are diseased. To be able to do this, the problem of muscle actions during rotations of the eye should be approached by way of consideration of the mechanics of the movement, which require that if the globe is to stay in any given position then the sum of the moments around the centre of rotation must be zero in that position. Robinson (1975) has formulated a model which incorporates this mechanical constraint, but to do so he had to make a number of more or less justifiable assumptions which are described in the next section.

THEORY

A complete description of the model is given in Robinson (1975) and what follows here will consist only of a statement of the main assumptions underlying his model so that they can be evaluated.

The first of these assumptions was that the origins and insertions of the muscles in the normal eye are adequately described by the data of Volkmann (1869) who used a co-ordinate system with the origin placed at the centre of rotation of the eye, which he judged to be 1.29 mm posterior to the geometric centre of the eye. If one shifts his origin forward by 1.29 mm along the primary direction of the line of fixation, one makes his co-ordinates directly comparable with those of Ruete and Fick, cited in Helmholtz (1911). This has been done by Von Kries and the results are given in an appendix in Helmholtz (1911). One may test whether or not the insertions of the muscles are consistent with the concept of a spherical globe by calculating the distances between the points of insertion of the muscles and the centre of rotation of the eye, which should all be equal with a spherical globe. In terms of the model, this corresponds to calculating the lengths of the insertion vectors of the muscles and the results of such a calculation are shown in Table 1. Considering the difficulty of making the measurements, the agreement is reasonable, although in order to set up the model it was assumed that the centre of rotation and geometrical centre of the eye are identical, which is not usually true.

The next two assumptions were concerned with specifying the shape of each muscle in any given position of the globe. The second assumption specified how the muscle was placed in relation to its insertion, a problem which is complicated by the fact that the muscles fan out at their insertion. Previous investigators had always selected the obvious assumption of the shortest path despite the mechanical restrictions at the insertions, but in this case it was proposed that the actual path lies somewhere between the shortest path and the path perpendicular to the line of insertion. Two criteria were outlined that should be satisfied by a reasonable assumption as to the angle of twist away from the perpendicular path. The first of these was that if the line of insertion stays perpendicular to the primary plane of the muscle, then the twist angle should be zero. This limits the

Table 1. Lengths of the insertion vectors of the extraocular muscles (mm) according to the various investigators

	LR	MR	SR	IR	SO	IO
REUTE	11.9	11.6	11.7	11.8	11.6	12.0
FICK	12.0	12.0	12.0	12.0	11.3	12.0
VOLKMANN (AFTER VON KRIES)	11.4	12.3	12.3	12.3	12.8	12.1
VOLKMANN (AFTER KREWSON)	12.0	13.1	13.0	13.0	12.2	11.3

path of each eye muscle as the direction of the insertion vector becomes directly opposite to the direction of the origin vector, whereupon slight movements of the eye cause extreme changes in the shortest path. The second criterion was that the twist angle should depend on the sideways force at the insertion. A satisfactory assumption was made by letting the twist angle depend on the cosine of the angle between the vector along the line of insertion of the muscle and the vector to its origin. In the primary plane of the muscle, this function is always zero and so there is no twist at the insertion.

The third assumption specified the path of the muscle away from its insertion. This assumption was directed towards ensuring that there is no abrupt change of direction when the muscle leaves the eyeball. This was achieved by assuming that the path of the muscle over the globe lay in a plane containing the vector corresponding to the direction in which the muscle leaves its insertion and the origin of the muscle. The intersection of this plane with the spherical globe is a circle, so that this assumption implies that the muscle makes contact with the globe along an arc of a circle.

As well as specifying the shapes of the muscles it is also necessary to specify the forces that they exert in the different orientations of the eye and the fourth and fifth assumptions were concerned with this aspect of the problem. The steady-state force exerted by a muscle is a function of its length and its innervation level. Innervation cannot be measured directly but it can be manipulated by asking a patient undergoing extraocular muscle surgery on the horizontal recti of one eye to look with the other eye at targets located in the horizontal plane at known angles with the primary direction of the line of fixation and measuring the force changes in the detached recti of the eye being operated on. The fourth assumption, then, consisted of a function describing the force exerted by a muscle in accordance with its length and its innervation which was based on the experimental data of Collins and O'Meara cited in Robinson (1975). It was found that if muscle tension was plotted against extension (ΔL), calculated as a percentage of the length in the primary position, then the function was a portion of an hyperbola. Furthermore, the effect of a change in the innervation level was to shift the curve along the muscle extension axis and this shift could be characterized by incorporating a factor (E) to reflect the level of innervation. The equation actually specified by Robinson (1975), after substitution of parameters, takes the form:

$$\text{Force (g)} = 0.9 \times (\Delta L + E) + \sqrt{38.94 + 0.81 \times (\Delta L + E)^2}.$$

The fifth assumption was that the force functions of the other muscles were identical to that of the lateral rectus, except for a multiplicative factor corresponding to their cross-sectional area, relative to that of the lateral rectus. The actual values for this factor were based on the data of Volkmann (1869) and were as follows:

LR	MR	SR	IR	SO	IO
1.0	1.04	0.68	0.95	0.5	0.47

As well as the active forces of the muscles, there are also passive forces due to check ligaments and other orbital structures which restrain the eye in movements away from the primary position. A function was developed to describe the way in which the passive force varied with the angle (beta) between the primary position and the line of fixation, based on the experimental results of Robinson *et al.* (1969) and Scott (1971). When the angle beta is given in degrees, the function specified by Robinson (1975), with the parameters inserted, takes the form:

$$\text{Passive force (g)} = 0.48 \times \text{beta} + 0.000156 \times \text{beta}^3.$$

The sixth assumption was that the passive force in any position could be described by this function and acted around the axis specified by Listing's law. A constant moment was added which made the resting point deviate 7.5° temporally which is consistent with the abduction seen in deep anaesthesia.

Given these assumptions, the problem of simulating extraocular muscle co-operation breaks down into two halves, which can be referred to as the innervation problem and the position problem. The innervation problem arises when the position of the eye is given and one has to determine the appropriate levels of innervation for each of the muscles. This involves finding the innervation values which result in the overall moment on the eyeball in that position being zero. Since there are six muscles and only 3 df for the globe, if each muscle is independently innervated there will be an infinite number of solutions to this problem. Hence, the law of reciprocal innervation was invoked and the seventh assumption was made, namely, that the innervation of the antagonist muscle was reciprocal to that of the agonist muscle for each of the three muscle pairs. The actual equation specifying the innervation of the antagonist in terms of that of the agonist as given in Robinson (1975) becomes, after insertion of the parameter values:

$$E(\text{antagonist}) = \{187.69/[E(\text{agonist}) + 9.7]\} - 9.7.$$

The position problem arises when one has determined the innervation values, but does not know what position the globe will take up to achieve mechanical equilibrium. Up to this point it has been assumed that the eye rotates in accordance with Listing's law, but with diseased eyes this need no longer be so. Therefore the final assumption involved setting up an additional passive force, governed by the same function as the original passive force, except that the torsion angle was substituted for the angle of deviation from the primary position, which opposed any torsional movements of the eye. This allowed some torsion in diseased eyes, but resulted in zero torsion in normal eyes.

RESULTS

Action of the individual muscles

Given the parameters of the model one can use it to gain some idea of the actual forces exerted by each of the muscles in any particular direction of gaze. To do this, the force exerted by each muscle is calculated by inserting the values for the extension and innervation of the muscle into the force equation specified earlier. Then, as was done by Krewson (1950) the axis around which the force of the muscle acts may be decomposed into an eye-centred system of Cartesian axes to obtain the relative amounts of the muscle force devoted to the various types of movement. These calculations have been done for nine central gaze positions, and the results are shown in Figs 1 – 3. It must be emphasized that the results will only be valid over this limited range of movements.

With respect to forces acting around the adduction/abduction axis (shown in Fig. 1), there are three points which are noteworthy. The first is that the horizontal recti develop the main forces, with the lateral rectus exerting the largest force of any muscle for any type of rotation. The second is that the vertical recti always adduct. The third point is that the obliques do not contribute anything significant to movements of abduction and adduction. Recordings of the actual muscle tensions in the lateral and medial recti during unrestrained eye movements of patients with strabismus have been made by Collins *et al.* (1975) and the agreement of the model with their results is good. They found that the minimum tension of each of the horizontal recti did not normally fall below 8 – 12 g and that the minimum tension of each muscle usually occurred 15° out of their field of action. These findings are matched by the predictions of the model, except for the location of the minimum tension of the medial rectus, which, as can be seen in Fig. 1, achieves its minimum tension in the primary position, instead of with 15° of abduction.

As regards the forces acting around the elevation/depression axis (shown in Fig. 2), it was found that the superior rectus exerts the dominant force in elevation and the inferior rectus exerts the dominant force in depression. However, the relative participation of the vertical recti and the obliques does change in accordance with the classical picture with the superior rectus acting more in abduction and the inferior oblique acting more in adduction. The model was also tried using the shortest-path assumption and the only consistent difference was found in connection with this component of the force, since it was found that with the shortest-path assumption both the horizontal recti elevated the eye with elevated gaze and depressed it with depressed gaze.

The torsional forces (shown in Fig. 3) are of especial interest since any imbalance in them leads to a deviation from Listing's law. Perhaps the most striking feature of these forces is that each muscle pair exerts opposing forces about the line of fixation, so that the muscle actions are predisposed towards a zero torsion equilibrium. Overall, the forces are smaller than for the other rotations so any passive moment about the axis will provide effective constraint against torsional movements.

Concerning the actions of the individual muscles, the forces exerted are as formulated in the classical description with the superior rectus and oblique both acting as intorters while the inferior rectus and oblique both act as extorters. Surprisingly, though, the horizontal recti produce a not inconsiderable amount of torsion. This result must be placed within the context of the assumption of the relative strengths of the muscles, for although they may project less than the other muscles onto the torsion axis, the horizontal recti counter this factor by exerting larger forces. This type of consideration will be overlooked by the purely geometrical analyses of Boeder (1962) and Solomons (1978).

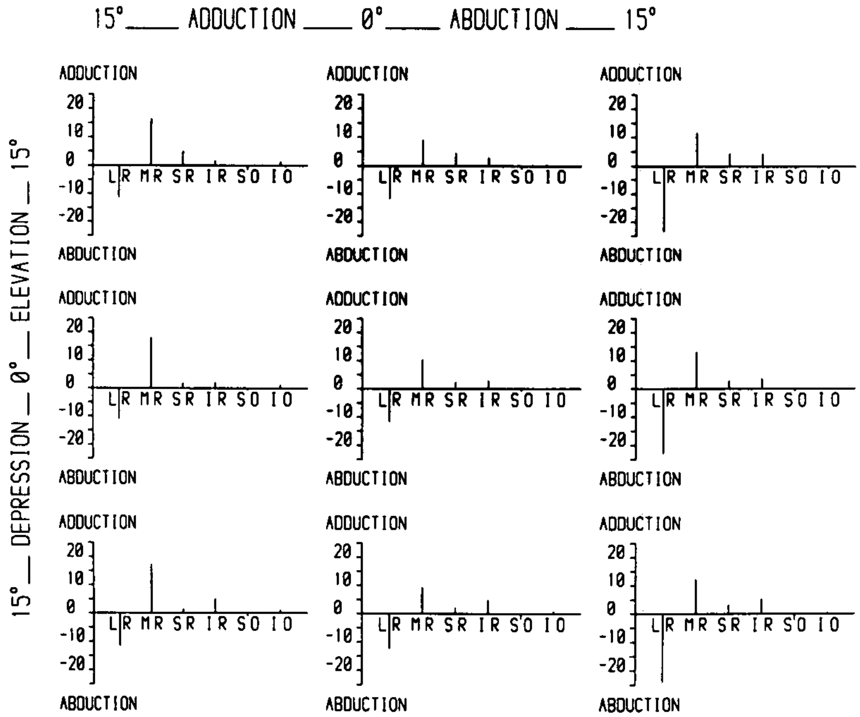

Fig. 1. Forces exerted by each of the muscles around the adduction – abduction axis in nine positions of gaze. For this and the following two graphs the conventions are as follows. The forces in each position are represented by a separate graph. The forces of individual muscles are distinguished by the letters LR for lateral rectus, MR for medial rectus, SR for superior rectus, IR for inferior rectus, SO for superior oblique, and IO for inferior oblique. The vertical axis of each graph gives the direction in which the muscle force acts and is calibrated in grams.

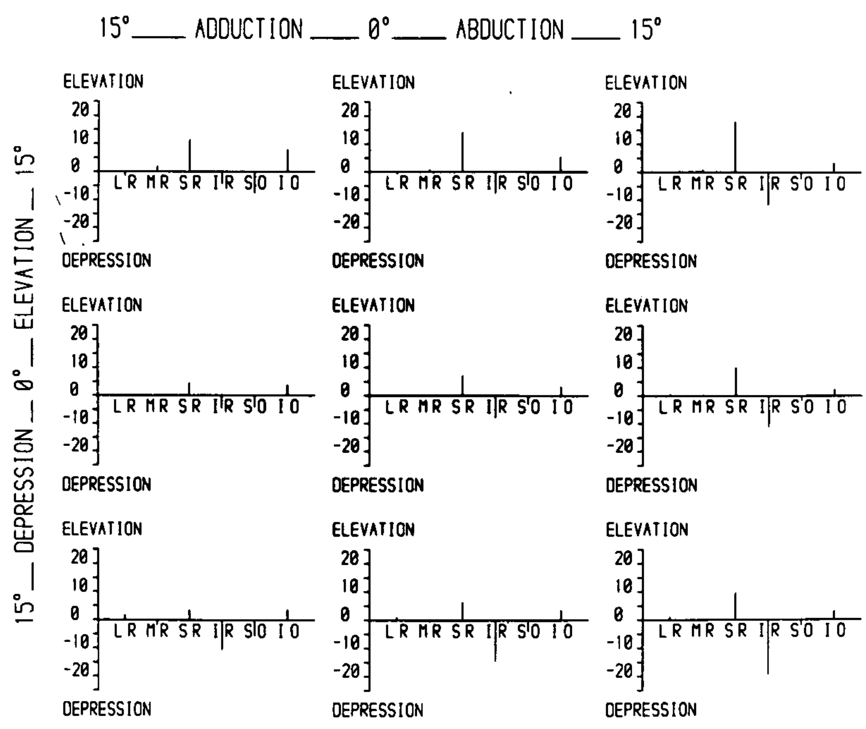

Fig. 2. Forces exerted by each of the muscles around the elevation – depression axis in nine positions of gaze.

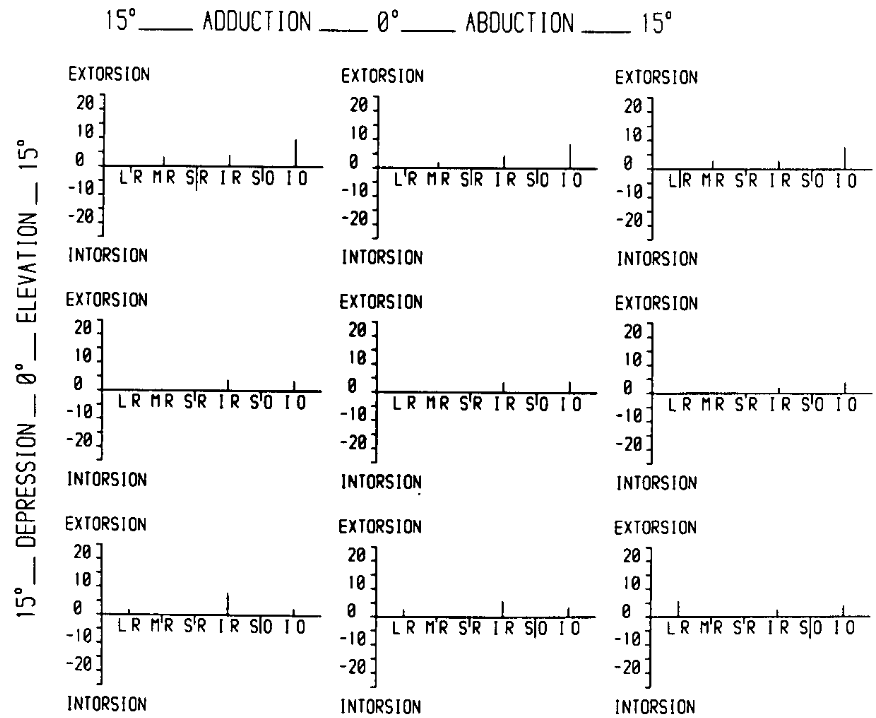

Fig. 3. Forces exerted by each of the muscles around the torsion axis in nine positions of gaze.

Effects of muscle paresis

Following the lead of France and Burbank (1979), the model has been used to simulate the effects of oculomotor nerve palsies. The model, as it stands, is essentially monocular, but by using two versions of the model in combination, the effect of lesions of individual oculomotor nerves of the right eye and the resulting Hess screen projections could be determined.

The Hess chart for the right eye shows the position adopted by the right eye when the left eye is fixating. In terms of the model, this involves solving the position problem for the right eye, given the normal innervation values. The chart for the left eye shows the position adopted to the left eye when the right eye is fixating. In terms of the model this involves first computing the innervation values needed to maintain the fixation of the affected right eye and then solving the position problem for the normal eye, given these innervation values.

The effects of damage to the third, fourth and sixth nerves are shown in Figs 4 – 6 respectively. The palsies were modelled by reducing the innervation level to the muscle or muscles supplied by the nerve to half their normal levels. These projections are reasonably consistent with those found in actual isolated nerve lesions. The numbers at each position in the figures give the predicted angle of torsion in degrees, with a positive number signifying a clockwise rotation about the line of fixation.

In order to investigate the role of the final assumption of a counter-torsional force, which was introduced to control the deviation from Listing's law in clinical conditions, the Hess chart corresponding to damage to the fourth nerve, which shows the most torsion, was repeated with the counter-torsional force halved and with it doubled. These changes made no appreciable difference to the shape of the resulting Hess chart, but with

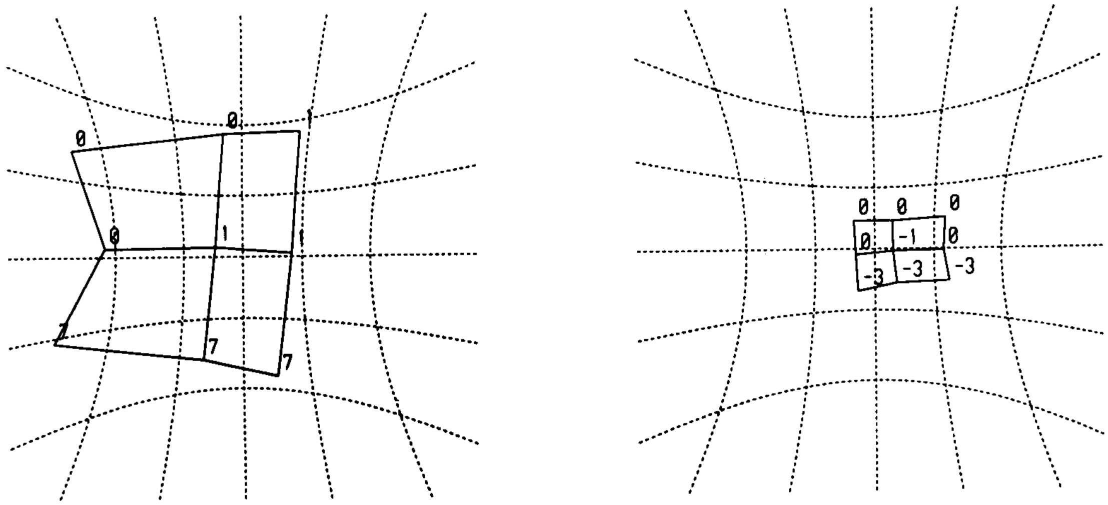

Fig. 4. Simulated Hess chart with damage to the third nerve. In this and all subsequent figures, the left half gives the Hess screen projection of the left eye and the right half gives the Hess screen projection of the right eye.

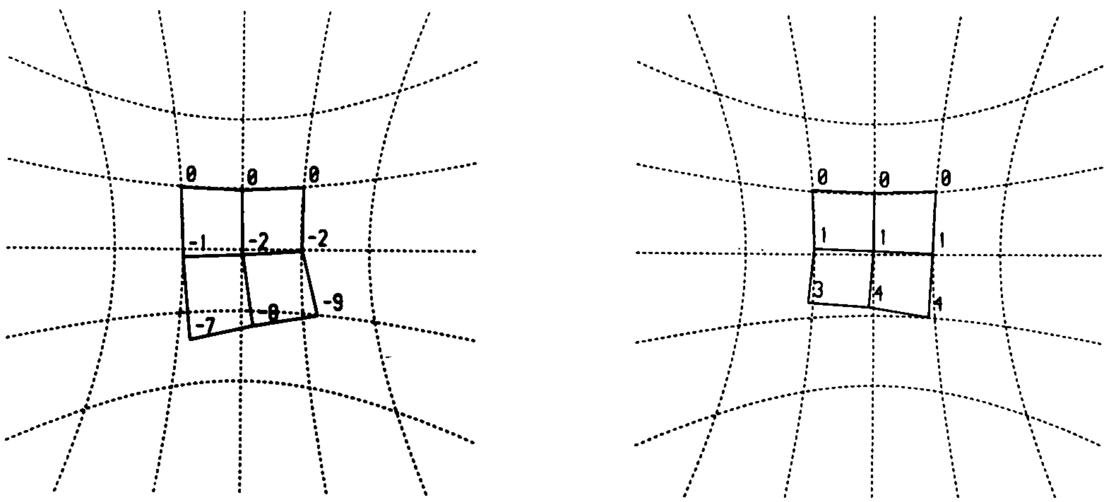

Fig. 5. Simulated Hess chart with damage to the fourth nerve.

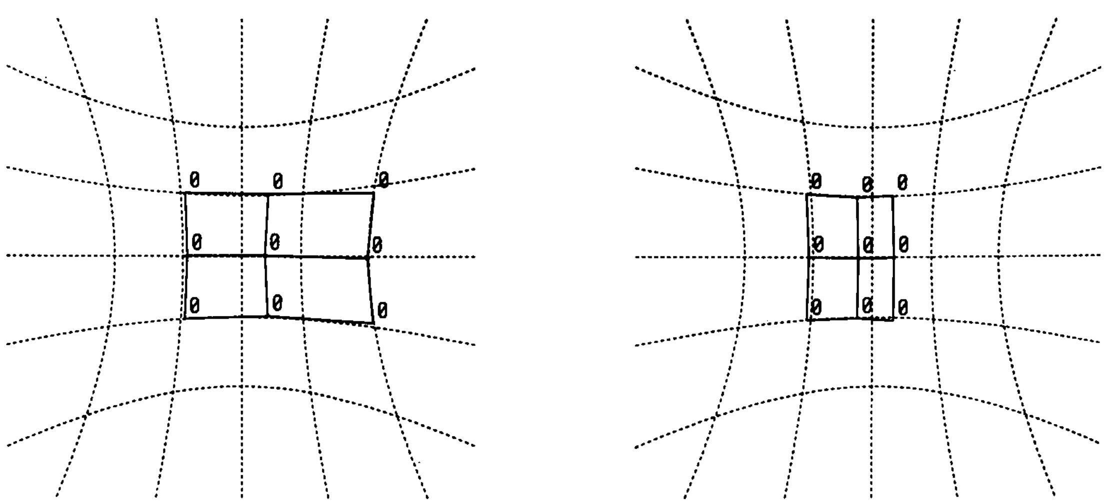

Fig. 6. Simulated Hess chart with damage to the sixth nerve.

a reduced counter-torsional force, more torsion occurred and with an increased counter-torsional force, less torsion occurred. The change in torsion in both cases was not large, being around 1° and occurring in the depressed-gaze positions. These results are closely related to the characterization of the force exerted by each muscle as being in part due to its extension and in part due to its innervation. For instance, if one considered the example of the sixth nerve lesion then it is noticeable that the movements to the left are relatively unaffected, because the force exerted by the lateral rectus of the right eye in these positions of gaze is mainly due to extension of the muscle rather than its level of innervation. Instead of altering the levels of innervation one could alter the muscle strength factor, to reproduce the effect of a diseased muscle as opposed to a diseased nerve. This has been done for the superior oblique and lateral rectus of the right eye using the same 50% reduction as with the nerve lesions and the results are shown in Figs 7 and 8 respectively.

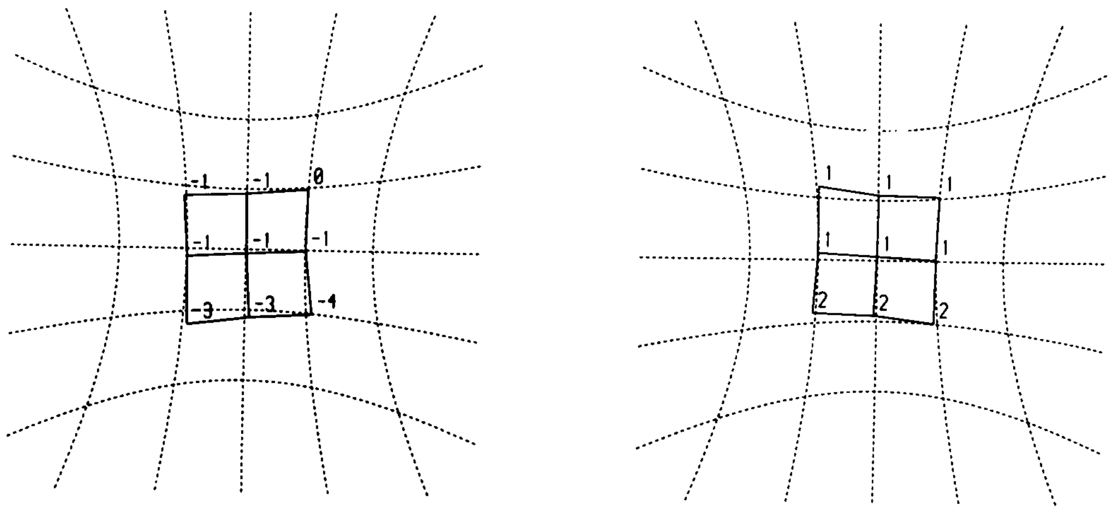

Fig. 7. Simulated Hess chart with paresis of the superior oblique.

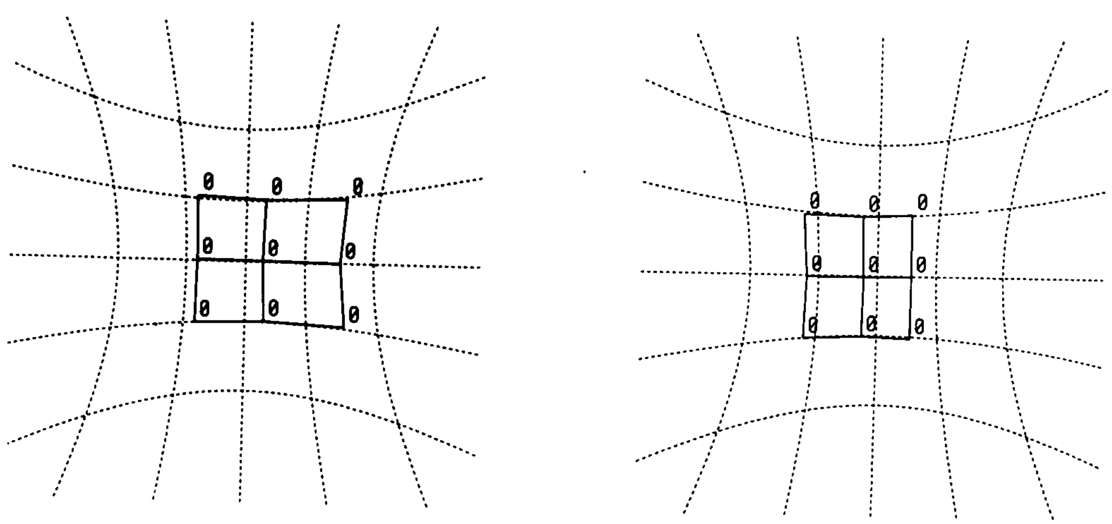

Fig. 8. Simulated Hess chart with paresis of the lateral rectus.

CONCLUSION

Overall, the model seems to provide a promising approach to understanding the mechanisms underlying some forms of squint. However, it is clear that it rests on a number of assumptions, which must be kept in mind if it is not going to be misleading. Obviously the assumptions are not equally valid and in order to isolate the more tentative ones an attempt has been made to assess the relative soundness of the assumptions.

The muscle insertions are not consistent with a spherical eyeball and it would be preferable if they were scaled so that the insertions were all the same distance from the centre of the eye, since the calculations of the paths of the muscles over the globe are based on the geometry of a spherical eye. In general, the model seems fairly robust with respect to the assumptions about the positions and shapes of the muscles, as demonstrated by the limited effects of switching to the shortest-path assumption.

The fourth assumption of the equation governing the relationship between the force exerted by the muscle and its length change and innervation, and also the sixth assumption of the passive-force equation are both based directly on experimental investigations and need only be changed to incorporate additional experimental results. The fifth assumption of the relative muscle strengths is a dominant one in that changes in this assumption will significantly alter the simulations produced by the model. Since it is based on the anatomical measurements of Volkmann (1869) rather than on actual measurements of relative force, it should be treated with caution.

On a methodological level, the question arises as to how much confidence one can put in the solution to the innervation problem. Fry (1978) has emphasized the point that an infinite variety of patterns of tension could be holding the eye in any given position, and whilst the reciprocal innervation assumption leads to unique solution, its formulation may not be correct in detail. This question is also pertinent to the origin of the tendency to adhere to Listing's law, which may be due to neural constraints on the pattern of innervation, but which in the model requires the assumption of a counter-torsional force. Fortunately it was found that alterations to the size of the assumed counter-rotational force did not markedly affect the positions adopted by the eye, only its angle of torsion, so the results produced by the model are relatively independent of this assumption.

As regards the future of such computer models, whilst there are enough parameters in the model for it to be flexible enough to simulate several types of squint and their surgical treatment, it will not be predictive until the parameter changes corresponding to such modifications as palsy, contracture, recession and resection have been isolated. Even as it stands, however, it provides a useful embodiment of much of our current knowledge of the actions of the extraocular muscles and its purely educational value pointed out by Robinson (1975) should not be overlooked.

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